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CREEP RESPONSE OF A GENERALIZED MAXWELL MODEL

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A b s t r a c t. A numerical method for determining the creep response of a generalized Maxwell model was developed. It was found that this response can be determined numerically. The procedure for determining the creep response also gives the stress response of each element in the generalized model. These stresses in the elements can be used to explain creep failure in biological materials.

K e y w o r d s: creep response, Maxwell model

INTRODUCTION

Biological materials such as fruits, vegetables, solid foods, tree branches, and hay cubes are viscoelastic solids. The behaviour of such materials can be modeled by linear viscoelastic models [8]. Researchers have successfully used the generalized Maxwell model to analyze stress-relaxation responses of apples [2,4], pears [1], potatoes [5], soybeans [6], rough rice [7], rice seedlings [10], and extruded durum semolina [3]. The generalized Maxwell model was often used to model stress relaxation behavior because it is easier to develop the analytical equation which governs the stress response of this model when the strain history is prescribed. However, this model is not commonly used for modeling the creep response, because the difficulty in obtaining the analytical equation for the strain associated with a prescribed stress. In cases where the stress history is prescribed, such as the creep response, it is more convenient to use the generalized Kelvin model.

Although theoretically there exists an equivalent generalized Kelvin model for a given generalized Maxwell model, and vise versa, such equivalent models can be obtained analytically only for the very simple cases. In more general cases, when the number of elements is more than four, it is practically impossible to obtain the equivalent models analytically. The generalized Maxwell model would be more useful for analyzing biological materials if its creep response can be determined.

The objective of this study is to develop a numerical method for determining the response of generalized Maxwell model.

THEORY

The generalized Maxwell model consists of a number of simple Maxwell models connected in parallel arrangement (Fig. 1).

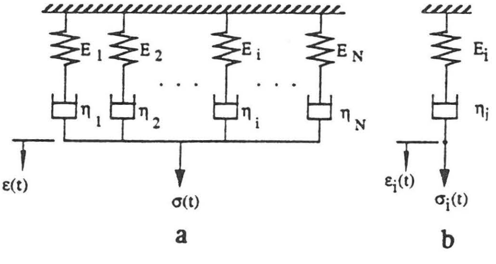

Fig. 1. Generalized Maxwell model (a) and the ith simple Maxwell element (b).

For this model, the stress $\delta(t)$ can be expressed as:

$$\sigma(t) = \sum_{i=1}^{N} \sigma_i(t) \qquad (1)$$

where $\sigma_i(t)$ is the ith Maxwell element, and the strain

$$\varepsilon(t) = \varepsilon_1(t) = \varepsilon_2(t) = \ldots = \varepsilon_i(t) = \varepsilon_N(t) \qquad (2)$$

The relationship between the stress $\sigma_i(t)$ and strain $\varepsilon_i(t)$ in each Maxwell element can be expressed as:

$$\dot{\sigma}_i(t) + \frac{E_i}{\eta_i}\sigma_i(t) = E_i\,\dot{\varepsilon}(t) \qquad (3)$$

Equation (3) can be rewritten as:

$$\sigma_i(t) = \frac{D}{\dfrac{D}{E_i} + \dfrac{1}{\eta_i}}\,\varepsilon(t) \qquad (4)$$

where D is the differential operator with respect to time, $D=d/dt$. The total stress can be expressed as:

$$\sigma(t) = \sum_{i=1}^{N} \sigma_i(t) = \left(\sum_{i=1}^{N} \frac{D}{\dfrac{D}{E_i} + \dfrac{1}{\eta_i}}\right)\varepsilon(t) \qquad (5)$$

By multiplying both sides of Eq. (5) by the product $\prod_{i=1}^{N}\left(\dfrac{D}{E_i} + \dfrac{1}{\eta_i}\right)$ and performing simple algebraic manipulations, one can obtain the following general different equation for the generalized Maxwell model:

$$\left[\left(\frac{D}{E_1} + \frac{1}{\eta_1}\right)\left(\frac{D}{E_2} + \frac{1}{\eta_2}\right)\left(\frac{D}{E_3} + \frac{1}{\eta_3}\right)\cdots\right]\sigma(t)$$
$$= \left[D\left(\frac{D}{E_2} + \frac{1}{\eta_2}\right)\left(\frac{D}{E_3} + \frac{1}{\eta_3}\right)\ldots + D\left(\frac{D}{E_1} + \frac{1}{\eta_1}\right)\left(\frac{D}{E_3} + \frac{1}{\eta_3}\right)\ldots + \ldots\right]\varepsilon(t) \qquad (6)$$

Equation (6) is difficult to solve analytically. However, for a prescribed strain input, the stress response of a generalized Maxwell model can be obtained, instead of solving Eq. (5), by first solving Eq. (3) for $\sigma_i(t)$ and using Eq. (1) to obtain $\sigma(t)$. For example, for a constant stain input ε_0 and known initial condition $\sigma_i(t_1)$ at t_1, the solution of Eq. (3) is:

$$\sigma_i(t) = \sigma_i(t)\,e^{-\frac{E_i}{\eta_i}(t - t_1)} \qquad (7)$$

and the stress relaxation of the generalized Maxwell model can be expressed as:

$$\sigma(t) = \sum_{i=1}^{N} \sigma_i(t_1)\,e^{-\frac{E_i}{\eta_i}(t - t_1)} \qquad (8)$$

On the other hand, when a prescribed stress input is given (e.g., creep loading with a constant stress, σ_0), it is impossible to obtain the strain response of the generalized Maxwell. In such cases, numerical method can be used to obtain the solution.

Numerical method

Given a prescribed constant stress σ_0 and $\varepsilon(t_{n-1})$, the strain at t_{n-1}, corresponding to the stress σ_0, the value of $\varepsilon(t_n)$, where $t_n - t_{n-1} = \Delta t$, can be determined numerically as follows (Fig. 2):

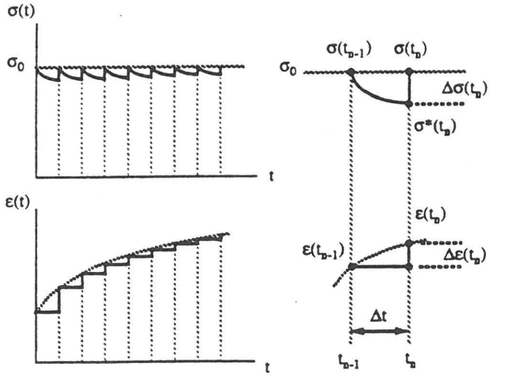

Fig. 2. Incremental changes of stress and strain in each discrete time interval.

Step 1: Assume that $\varepsilon(t)$ is constant (i.e., $\varepsilon(t)=\varepsilon(t_{n-1})$ within Δt, then use Eq. (7) to calculate $\sigma_i{}^*(t_n)$:

$$\sigma_i{}^*(t_n) = \sigma_i(t_{n-1})\, e^{-\frac{E_i}{\eta_i}(\Delta t)} \tag{9}$$

and

$$\sigma^*(t_n) = \sum_{i=1}^{N} \sigma_i(t_{n-1}) e^{-\frac{E_i}{\eta_i}(\Delta t)} \tag{10}$$

Step 2: Determine $\Delta\delta(t_n)$, the difference between the prescribed σ_0 and $\sigma^*(t_n)$:

$$\Delta\sigma(t_n) = \sigma_0 - \sigma^*(t_n)$$

Step 3: Determine $\varepsilon(t_n)$ corresponding to σ_0:

$$\Delta\varepsilon(t_n) = \Delta\sigma(t_n) \Big/ \sum_{i=1}^{N} E_i$$

$$\varepsilon(t_n) = \varepsilon(t_{n-1}) + \Delta\varepsilon(t_n)$$

Step 4: Determine $\sigma_i(t_n)$:

$$\Delta\sigma_i(t_n) = \Delta\varepsilon(t_n) E_i$$

$$\sigma_i(t_n) = \sigma_i{}^*(t_n) + \Delta\sigma_i(t_n)$$

Step 5: Increment n and repeat steps 1 to 4 until t reaches the desired value, t_{max}. The above procedure yields a set of discrete points of the strain response:

$$\varepsilon(t_n), n = 1, 2, \ldots, t_{max}/\Delta t$$

and a set of discrete points of the stress response in each ith Maxwell element:

$$\sigma_i(t_n), i = 1, 2, \ldots, N$$

the sum of which equals the prescribed stress σ_0.

A computer program has been written to perform these numerical calculations. The program is available to the readers upon request.

Numerical example

In the following example, we used experimental data obtained from a stress relaxation test made on a Barlett pear [1]. The parameters E_i and n_i, i =1,2, and 3 for the generalized Maxwell model are given in Table 1. These parameters were used to determine the creep response of the same generalized Maxwell model under a constant stress load. Figure 3 shows the creep response, $\varepsilon(t)$, of this generalized Maxwell model under a constant stressload of 0.7 MPa, and Fig. 4. shows the stress $\sigma(t)$ in each simple Maxwell element and the prescribed stress load, σ_0, which is equal to the sum of $\sigma_i(t)$.

Table 1. Viscoelastic parameters of a generalized model that represents the tissue of a Bartlett pear

i	E_i (MPa)	η_i (MPa s)
1	4.039	6213.14014
2	0.641	7.049
3	2.309	2.859

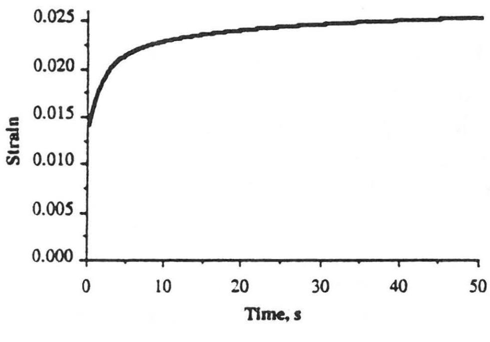

Fig. 3. Numerically determined creep response of a generalized Maxwell model using the viscoelastic parameters in Table 1. The model was subjected to a constant stress load of σ_0.

Since the fruit tissue consists of different components - individual cells and intercellular liquids and gases, a load applied to the tissue is carried by the pressure in the cells and in the intercellular fluids. However, since the intercellular fluids can flow

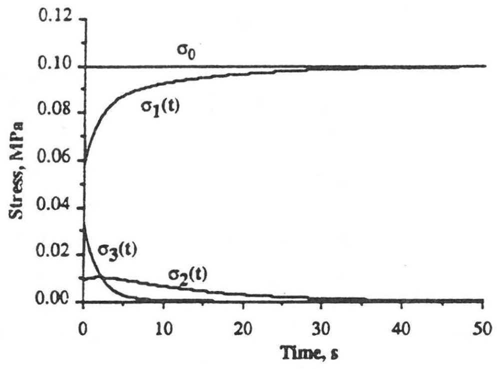

Fig. 4. Numerically determined stress response of each element of a generalized Maxwell model using the viscoelastic parameters in Table 1. The model was subjected to a constant stress load of σ_0.

more freely from a high-pressure region to a low-pressure region, the portions of the load that is taken up by the fluids will diminish with time. The load that is carried by the gas pressure will decrease more rapidly than that carried by the liquid. A generalized Maxwell model consists of a number of simple Maxwell elements with different relaxation time constant (η_i/E_i). The element with short time constant represents such component as the intercellular gases which can sustain a load for only very short period of time. The element with medium time constant may represent the intercellular liquids which flow at a slower rate than gases. The cells, as a whole, are represented by the element with a very long relaxation time constant, because fluids inside the cells can only move in and out of the cells at a very slow rate. Figure 4 shows that when a fruit is subjected to a constant load, the stress exerted on each component of the tissue continues to change with time. Segerlind and Dal Fabbro [9] found that it is possible to cause creep failure in apple tissue. They applied a critical constant stress load on a tissue specimen and detected cell rupture some time after the load was applied (during creep).This can be explained by the gradual increase of σ_1 (t) with time in Fig. 4.

CONCLUSION

The creep response of a generalized Maxwell model can be determined numerically. The procedure for determining the creep response also gives the stress response of each element in the generalized model. These stresses in the elements can be used to explain creep failure in biolgical materials.

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